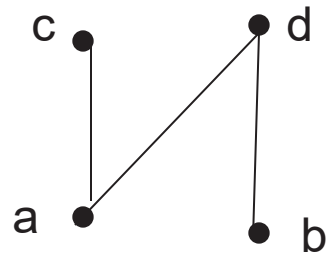
Extremal Elements: Example 1



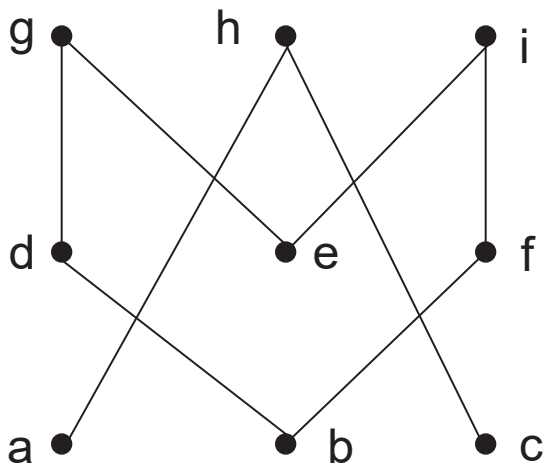
What are the minimal, maximal, elements?

- Minimal: {a,b}
- Maximal: {c,d}
-

Extremal Elements: Example 2

Give lower/upper bounds & glb/lub of the sets:

{a,c} and {b,d}



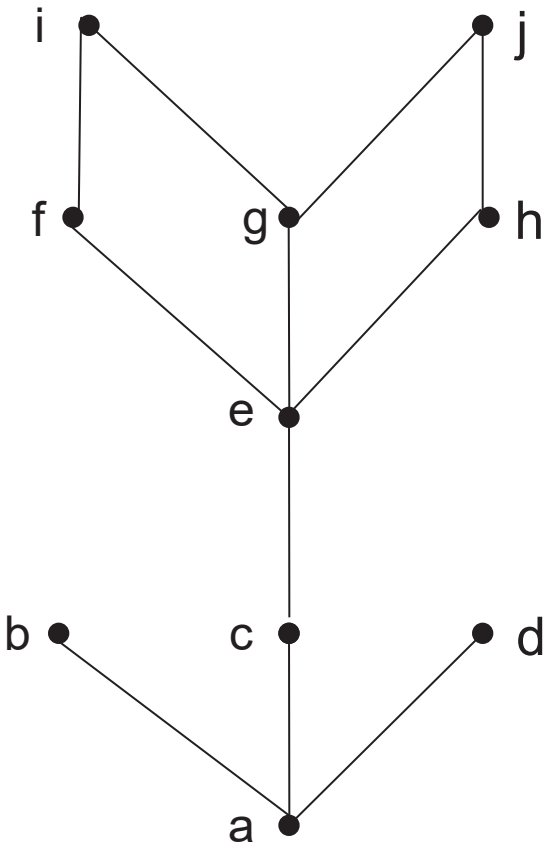
{a,c}

- Lower bounds: ∅, thus no glb
- Upper bounds: {h}, lub: h

{b,d}

- Lower bounds: {b}, glb: b
- Upper bounds: {d,g}, lub: d because $d \prec g$

Extremal Elements: Example 3



- Minimal/Maximal elements?
  - Minimal & Minimum element: a
  - Maximal elements: b,d,i,j
- Bounds, glb, lub of {c,e}?
  - glb is c
  - Upper bounds: {e,f,g,h,i,j}, thus lub is e
- Bounds, glb, lub of {b,i}?
  - glb is c
  - lub DNE

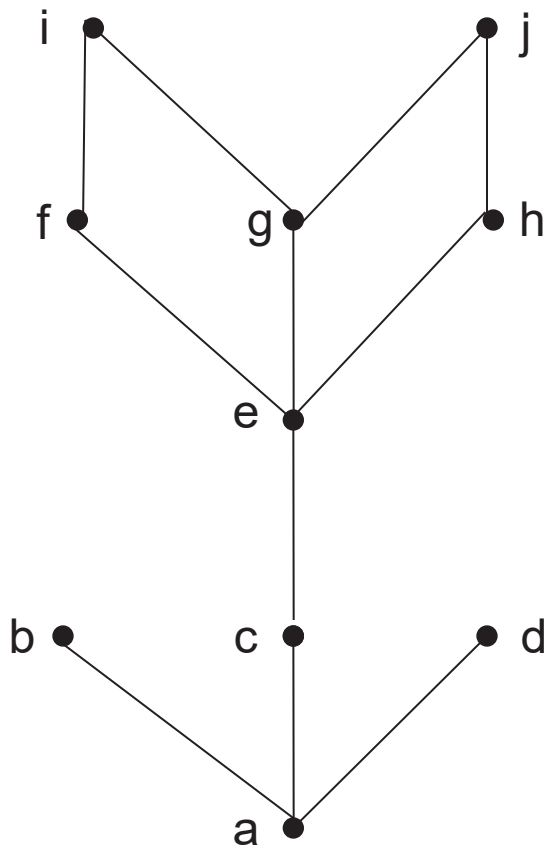
Lattices

- A special structure arises when every pair of elements in a poset has an lub and a glb
- **Definition**: A lattice is a partially ordered set in which every pair of elements has both
  - a least upper bound and
  - a greatest lower bound

Lattices: Example 1

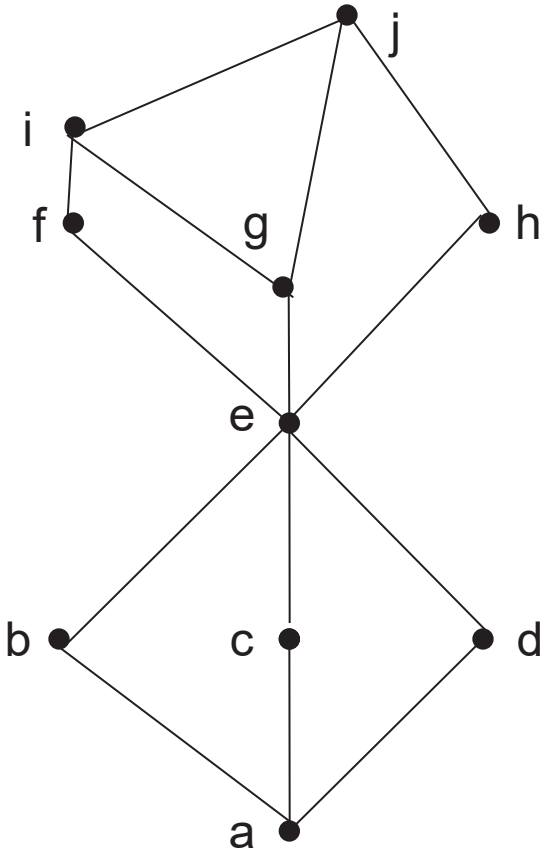
- Is the example from before a lattice?

- **No, because the pair {b,c} does not have a least upper bound**



Lattices: Example 2

- What if we modified it as shown here?



- **Yes, because for any pair, there is an lub & a glb**